

BGSU Mathematics Competition  
February 24th 2024 **B** (below Calculus II)

No cell phones are allowed. Show all your work. Justify your answers.

- (1) Solve for  $x \in \mathbb{R}$ .

$$(x^2 + x)^2 + |x^2 + x| - 2 = 0.$$

- (2) Solve for  $x \in \mathbb{R}$ .

$$\sqrt[x]{x} = \sqrt{x^x}$$

- (3) Simplify as much as possible the expression

$$E = 12(13^{12} + 13^{11} + 13^{10} + 13^9 + 13^8 + 13^7 + 13^6 + 13^5 + 13^4 + 13^3 + 13^2 + 14) + 1.$$

- (4) The numbers 1 to 50 are written in a row. Can the sign + and – be placed between them such that the value of the resulting expression is 0?

- (5) In a certain year there were exactly four Mondays and exactly four Fridays in January. In what day of the week did January 20th fall that year?

- (6) Consider a rubber ball that bounces. Suppose when the ball is dropped, on each bounce it returns to  $9/10$  of its previous height. So on the first bounce it returns to  $9/10$  its original high, on the second bounce to  $(9/10)^2$  its original height, etc. If the ball is dropped from an initial height of 3 meters, what will be the total distance the ball travels?

- (7) Prove that no matter how we choose 51 distinct natural numbers from the set

$$\{1, 2, 3, \dots, 99, 100\}$$

at least two of them must be consecutive.

- (8) A bag contains a number of marbles of which 78 are red, 24 are blue, and the rest are green. If the probability of selecting a green marble is  $1/3$ , what is the probability of selecting a red marble?

**Registration 2024 BGSU Mathematics Competition;**

**Your NAME:**

E-mail:

(Optional)

Math class you are taking this semester/year:

Name of your instructor(s):

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1)

2)

3)

4)

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Total: